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Cosmogenesis and Collapse

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Some possible benefits of dynamical collapse for a quantum theory of cosmogenesis are discussed. These are a possible long wait before creation begins, creation of energy and space, and choice of a particular universe out of a superposition.

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I. INTRODUCTION

Suppose one believes that the onset of our universe should be described by quantum theory. Quantum evolution famously produces a state vector which is a superposition of possibilities, but our particular universe is just one of those possibilities. How was the choice made? A dynamical collapse theory can help with this.

Such a theory as CSL (Continuous Spontaneous Localization) [1, 2], constructed to describe wave function collapse within the present universe, works quite well. The state vector evolves continuously, governed by a Hamilton whose Hermitian part is the usual Hamiltonian, and whose anti-hermitian part depends upon a randomly fluctuating field. When the hermitian Hamiltonian evolves the state vector into a superposition of macroscopically different states, the anti-hermitian Hamiltonian causes a rapid evolution to one of them.

A dynamical collapse theory posits that the state vector corresponds to the real state of the world, and provides a mechanism to accomplish this. If one believes that nature uses the collapse mechanism to choose the outcome of an everyday event, it is natural to suppose that a similar mechanism was at work for the Big Event.

Suppose the pre-universe is modeled by a state $|0\rangle$ which represents no space. Further, suppose that time exists. (An eventual hope might be to have time come out of such a theory, but crawling comes before walking.)

A Hamiltonian must be chosen to evolve the state $|0\rangle$ into our early universe. One might suppose this is a relatively rare event, i.e., the universe has a low probability/sec of being created out of the pre-universe. Dynamical collapse can help with this.

In standard quantum theory with the instantaneous collapse postulate, there is a phenomenon called the "Zeno" or "watched pot" effect. Suppose a system starts in state $|0\rangle$ and evolves under Hamiltonian H for a short time Δt, followed by instantaneous collapse. Suppose this occurs repeatedly over the time interval T. It is straightforward to show that the probability the system remains in state $|0\rangle$ is $\exp -[(\Delta H)^2 T\Delta t + \mathrm{o}(\Delta t)^2]$, where $(\Delta H)^2 \equiv \langle 0|H^2|0\rangle - \langle 0|H|0\rangle^2$. Thus, as Δt gets smaller, the probability that $|0\rangle$ evolves is reduced. A comparable behavior occurs for continuous collapse. When the collapse rate parameter λ increases sufficiently, the probability that the initial state vector evolves is reduced.

Not only space needs to be created, but also the energy which fills that space. Dynamical collapse can help with this.

In usual quantum theory, energy is conserved. However, that need not be the case in a dynamical collapse theory. For example, in CSL, particle wave functions are narrowed by the collapse process, which means that their energy grows[3].

In this paper, a simple cosmogenesis model to illustrate these features is presented. Its mathematics is that of a spin 1/2 caused to rotate with angular frequency ω by a constant y-directed magnetic field, and that of a perturbed harmonic oscillator, both subject to collapse dynamics.

The spin 1/2 acts like a gatekeeper to block (state $|\uparrow\rangle$) or allow (state $|\downarrow\rangle$) the harmonic oscillator to operate. The spin starts in the block state but is prevented from rotating with frequency ω because of "watched pot" behavior: a sufficiently large value of the collapse rate parameter λ can make the probability/sec of a transition to the allow state arbitrarily small.

The harmonic oscillator starts in the ground state $|0\rangle$ representing no space volume. The states $|n\rangle$ represent a universe of n cells, with energy ϵn and volume vn. To be concrete, we shall suppose that ϵ=Planck energy. v=Planck volume is a possible choice, or v may be an increasing function of time. One may consider that the energy generated is the dark energy alone, or all the energy: the nature of the energy and the size of v do not play a role in the dynamics (but see section VI).

Due to a perturbation, the oscillator would just oscillate between $|0\rangle$ and slightly higher $|n\rangle$ states. (Two kinds of perturbation shall be considered, an added term proportional to position in section III and to squared position in IV.) However, because of the attendant collapse (rate parameter λ'), the universe is generated: the state vector evolves states of increasing values of n. With the initial state $|\uparrow> |0\rangle$, the (unnormalized) state vector at time t is taken to be

$$|\Psi, t\rangle_{w,w'} = \mathcal{T} e^{-\int_0^t dt'\{\frac{i}{2}\omega\sigma_2 + \frac{1}{4\lambda}[w(t')-2\lambda\sigma_3]^2\}} \cdot e^{-\int_0^t dt'\{i[\epsilon a^\dagger a + g(a^\dagger + a)^s]Q + \frac{1}{4\lambda'}[w'(t')-2\lambda' a^\dagger a]^2\}} |\uparrow> |0\rangle. \quad (1)$$

In Eq.(1), $\mathcal{T}$ is the time-ordering operator. If there

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were no collapse, the spin would rotate clockwise in the $x-z$ plane. $Q \equiv |\downarrow\rangle\langle\downarrow|$ is the projection operator on the allow state. g is a coupling constant: we shall consider the cases of $s=1$ (an interaction which displaces the harmonic oscillator) and $s=2$ (an interaction which changes its spring constant).

$w(t')$ and $w'(t')$ are white noise random functions of time. The probability density for w, w' to take on particular values over the time interval $(0 \leq t' \leq t)$ is given by the probability rule:

$$P(w(t), w'(t)) \sim_{w,w'}\langle\Psi, t|\Psi, t\rangle_{w,w'} \tag{2}$$

where the proportionality constant is determined by

$$\int_{-\infty}^{\infty} DwDw'P = 1$$

with $DwDw' \equiv \prod_i dw(t_i)dw'(t_i)$ $(t_{i+1}-t_i \equiv dt)$.

According to Eq.(1), each different $w(t')$, $w'(t')$ gives rise to a different state vector $|\Psi, t\rangle_{w,w'}$. The probability of that state vector (of that $w(t')$, $w'(t')$) is given by Eq.(2). Eqs.(1,2) completely define the model. It only remains to draw the consequences.

In this paper, we shall only consider the evolutions of the spin and harmonic oscillator separately, not their joint evolution, in order to keep the discussion simple and to elucidate the features of each.

II. GATEKEEPER/SPIN

A numerical analysis has been given for this problem[4]. Here we give an analytical treatment.

The Fokker-Planck equation for the probability distribution $R(\theta, t)$ of the angle $0 \leq \theta \leq 2\pi$ that the spin makes with the z-axis is

$$\frac{\partial}{\partial t}R(\theta,t) = -\frac{\partial}{\partial\theta}\Big[\frac{\overline{d\theta}}{dt}R(\theta,t)\Big] + \frac{1}{2}\frac{\partial^2}{\partial\theta^2}\Big[\frac{\overline{(d\theta)^2}}{dt}R(\theta,t)\Big], \tag{3}$$

where the average is over all possible trajectories of θ due to all possible $w(t)$. Appendix A shows how to calculate the drift $\overline{d\theta}/dt$ and the diffusion $\overline{d\theta^2}/dt$ (Eqs.(A6,A7)), yielding the Foker-Planck equation:

$$\frac{\partial}{\partial t}R = -\frac{\partial}{\partial\theta}[\omega - 2\lambda\sin\theta\cos\theta]R + 2\lambda\frac{\partial^2}{\partial\theta^2}\sin^2\theta R. \tag{4}$$

Qualitatively, we see that the drift's steady clockwise rotation with angular velocity ω is opposed by the stochastic drift in the first and third quadrants, and aided in the second and fourth quadrants: the stochastic drift tends to drive the spin to the nearest pole $(\theta = 0, \pi)$. The diffusion rate is largest at $z=0$. The diffusion rate approaches 0 at the poles $\sim \theta^2, (\pi-\theta)^2$. This makes the poles absorbing boundaries[6]: if $\omega = 0$, all the trajectories eventually end up at the poles (collapse).

A. $\omega = 0$

We begin by considering uncontaminated collapse behavior ($\omega = 0$). Although Eq.(4) is exactly soluble (see [5], restated in Appendix B in the notation of this paper), we prefer to examine its properties in a way that is transferrable to the $\omega \neq 0$ case.

Here is the general context[5]. Consider an ensemble of normalized state vectors $|\psi, t\rangle = \sum_{k=1}^{N} c_k(t)|k\rangle$ which evolve stochastically, but start out identically at $t=0$. Two conditions on $x_k(t) \equiv |c_k(t)|^2$ are sufficient to obtain collapse behavior obeying the Born rule. These are the Martingale condition $\overline{x_k(t)} = x_k(0)$ and the asymptotic correlation condition $\overline{x_k(\infty)x_l(\infty)} = 0$ $(k \neq l)$.

The latter equation is an integral over the product of non-negative terms. The only way it can be satisfied is if, for every state, every $x_k(\infty)$ but one vanishes. Therefore, the asymptotic probability density distribution must be

$$P(x_1, ...x_n, \infty) = a_1\delta(1-x_1)\delta(x_2)...\delta(x_N) + ...$$

The Martingale condition then implies $a_k = x_k(0)$, i.e., the Born rule.

For our problem, there are two states: $x_1(t) = \cos^2[\theta(t)/2]$ is the squared amplitude of $|\uparrow\rangle$ and $x_2(t) = \sin^2[\theta(t)/2]$ is the squared amplitude of $|\downarrow\rangle$. We use (4) to calculate $\overline{\cos\theta}(t)$, $\overline{\sin^2\theta}(t)$. After integrations by parts, we obtain:

$$\frac{\partial}{\partial t}\overline{\cos\theta}(t) \equiv \frac{\partial}{\partial t}\int_0^{2\pi} d\theta\cos\theta R = 0 \tag{5}$$

$$\frac{\partial}{\partial t}\overline{\sin^2\theta}(t) \equiv \frac{\partial}{\partial t}\int_0^{2\pi} d\theta\sin^2\theta R = -4\lambda\overline{\sin^4\theta}(t). \tag{6}$$

It follows from Eq.(5) that the squared amplitude $(1/2)(1+\overline{\cos\theta}) = \overline{\cos^2(\theta/2)}$ is constant, so the Martingale condition is satisfied.

Now, $\overline{\sin^2\theta}(t)$ is non-negative but, according to Eq.(6), it keeps decreasing unless $\overline{\sin^4\theta}(t)$ vanishes. Therefore, $\overline{\sin^4\theta}(t)$ must vanish at some time $t=T$. But, that is only possible if $R(\theta, T)$ is non-zero only where $\sin^4\theta$ is zero, i.e., at $\theta = 0, \pi$. But, this means that $\overline{\sin^2\theta}(T) = 0$ too. Since the average of the product of the up and down amplitudes is $\overline{\sin^2[\theta/2]\cos^2[\theta/2]}(T) = (1/4)\overline{\sin^2\theta}(T) = 0$, the asymptotic correlation condition is therefore satisfied. Thus, there is collapse satisfying the Born condition.

B. $\omega \neq 0$

It follows from Eq.(4) that

$$\frac{\partial}{\partial t}\overline{\cos\theta}(t) = -\omega\overline{\sin\theta}(t) \tag{7}$$

$$\frac{\partial}{\partial t}\overline{\sin\theta}(t) = \omega\overline{\cos\theta}(t) - 2\lambda\overline{\sin\theta}(t), \tag{8}$$

whose solution is

$$\overline{\cos\theta}(t) = \frac{1}{\lambda_+ - \lambda_-}\Big\{ \cos\theta_0[\lambda_+ e^{-\lambda_- t} - \lambda_- e^{-\lambda_+ t}] - \omega\sin\theta_0[e^{-\lambda_- t} - e^{-\lambda_+ t}]\Big\}, \quad (9)$$

$$\lambda_\pm \equiv \lambda \pm \sqrt{\lambda^2 - \omega^2} \approx 2\lambda, \frac{\omega^2}{2\lambda} \quad \text{for} \quad \lambda >> \omega.$$

where θ_0 is the value of θ at $t = 0$.

Consider the initial state $\theta_0 = 0$, spin up. The ensemble average squared amplitude of $|\uparrow\rangle$ is $\overline{\cos^2[\theta/2]}(t) = .5[1 + \overline{\cos\theta}(t)]$. Thus, according to Eq.(9),

$$\overline{\cos^2[\theta/2]}(t) = \frac{1}{2(\lambda_+ - \lambda_-)}\Big\{\lambda_+[1 + e^{-\lambda_- t}] - \lambda_-[1 + e^{-\lambda_+ t}]\Big\} \approx \frac{1}{2}\Big[1 + e^{-(\omega^2/2\lambda)t}\Big]. \quad (10)$$

Eq.(10) shows the "watched pot" behavior mentioned earlier. For $\omega^2/\lambda << 1$, most trajectories remain in the neighborhood of $\theta = 0$, i.e., there is only a small probability for a transition from $|\uparrow\rangle$ to $|\downarrow\rangle$.

Although it is an irrelevant feature for our application of this model, we close by remarking that, eventually, an equilibrium distribution of spins is reached, which is independent of the original state. When $R(\theta, t)$ is taken to be independent of t, Eq.(4) provides the first order differential equation

$$-[\omega - 2\lambda\sin\theta\cos\theta]R_{eq}(\theta) + 2\lambda\frac{\partial}{\partial\theta}\sin^2\theta R_{eq}(\theta) = -C, \quad (11)$$

whose solution is

$$R_{eq}(\theta) = C\frac{1}{|\sin\theta|^3}\int_0^\infty dz \frac{e^{-\frac{\omega}{2\lambda}z}}{[1 + (z - \cot\theta)^2]^{3/2}} \quad (12)$$

According to Eq.(12), the equilibrium probability distribution is largest at $\theta = 0+, \pi+$, and falls off as θ increases from 0 to $\pi-$ and from $\pi+$ to $2\pi-$, with a step-up discontinuity crossing the poles in the clockwise direction.

III. SPACEGENERATOR/OSCILLATOR I

The state vector evolution we consider here is, from Eq.(1),

$$|\Psi, t\rangle_w = \mathcal{T} e^{-\int_0^t dt'\{iH + \frac{1}{4\lambda}[w(t') - 2\lambda N]^2\}}|0\rangle. \quad (13)$$

where $H = \epsilon N + g(a^\dagger + a)$ and $N \equiv a^\dagger a$. The density matrix follows from Eqs.(1,2):

$$\rho(t) = \int Dw\, {}_w\langle\Psi, t|\Psi, t\rangle_w \frac{|\Psi, t\rangle_w\, {}_w\langle\Psi, t|}{{}_w\langle\Psi, t|\Psi, t\rangle_w} = \mathcal{T} e^{-\int_0^t dt'\{i[H_L - H_R] + \frac{\lambda}{2}([N_L - N_R]^2\}}|0\rangle\langle 0|, \quad (14)$$

where an operator with subscript L (R) acts to the left (right) of ρ, and $\mathcal{T}$ time reverse-orders operators to the right of ρ. Therefore, from (14), the density matrix evolution equation is

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] - \frac{\lambda}{2}[N, [N, \rho(t)]]. \quad (15)$$

With the notation $\overline{\langle A\rangle(t)} \equiv \text{Trace}[A\rho(t)]$, we find the coupled equations

$$\frac{d\overline{\langle N\rangle(t)}}{dt} = ig[\overline{\langle a\rangle(t)} - \overline{\langle a^\dagger\rangle(t)}] \quad (16)$$

$$\frac{d\overline{\langle a\rangle(t)}}{dt} = -i\epsilon\overline{\langle a\rangle(t)} - ig - \frac{\lambda}{2}\overline{\langle a\rangle(t)}. \quad (17)$$

Solving Eqs.(16, 17) and the complex conjugate of (17) results in:

$$\overline{\langle N\rangle(t)} = \frac{2g^2}{(\lambda/2)^2 + \epsilon^2}\Big\{\frac{\lambda}{2}t - \cos\theta + e^{-\frac{\lambda}{2}t}\cos[\epsilon t - \theta]\Big\} \quad (18)$$

where $\tan\theta = \epsilon/\lambda$.

We see from (18) that, when $\lambda = 0$, there is only oscillation, but when $\lambda \neq 0$ there is linear growth of the ensemble mean of the expectation value of the energy ϵN and the volume vN. The mechanism is that the coupling $\sim g$ causes N to oscillate by a small amount away from its initial value 0, but there is a chance that collapse will occur to an $N \neq 0$ value, and then the oscillation takes it away from that, etc. (It is worth noting here that "watched pot" behavior is again in evidence, i.e., $\lambda \to \infty$ implies $\overline{\langle N\rangle} \to 0$.)

IV. SPACEGENERATOR/OSCILLATOR II

In the model in the previous section, the mean volume of space grows linearly with time. Although inflation models make space expand exponentially with time, they do not start from zero space as do the models under consideration here. Inflation models assume that enough space had been created earlier so that it may subsequently be described by classical general relativity, with the inflaton field which lives on it obeying quantum field theory.

There is nothing wrong with an initial space creation that grows linearly with time. Nonetheless, it is interesting to see that an exponential growth can be obtained with only a slight modification of the model of section III: replace the perturbation in the Hamiltonian by $g(a^\dagger + a)^2$.

One then obtains from Eq.(14) the coupled equations

$$\frac{d\overline{\langle N\rangle}}{dt} = 2ig[\overline{\langle a^2\rangle} - \overline{\langle a^{\dagger 2}\rangle}] \quad (19)$$

$$\frac{d\overline{\langle a^2\rangle}}{dt} = -2\overline{\langle a^2\rangle}[i\epsilon + 2ig + 2\lambda] - 2ig[1 + 2\overline{\langle N\rangle}] \quad (20)$$

and the complex conjugate of (20). Writing all three quantities as $\sim \exp st$ and putting that into the homogeneous part of the equations results in the cubic equation:

$$f(s) \equiv s^3 + 4\lambda s^2 + (\epsilon^2 + \lambda^2 + 4\epsilon g)s - 32\lambda g^2 = 0. \quad (21)$$

For example, if $g = 3\lambda/2$ and $\epsilon = [\sqrt{14} - 3]\lambda$, Eq.(21) becomes

$$[s - 2\lambda][s - 6\lambda e^{i2\pi/3}][(s - 6\lambda e^{-i2\pi/3}] = 0.$$

For another example, if $\lambda << g = \epsilon/4$, then there is a root for which the s^3 and s^2 terms may be neglected, with the remaining terms giving $s \approx 32\lambda g^2/(\epsilon^2 + 4\epsilon g) = \lambda$.

Generally, it follows from Eq.(21) that there are two complex conjugate roots with negative real parts. Because $f(0) = -32\lambda g^2 < 0$ and f has positive slope for $s \geq 0$, there is one solution of (21) which is a positive real root, giving rise to an exponential growth of the mean energy and volume of space.

V. ON CHOOSING THE UNIVERSE

As an indication of how the collapse acts to "choose a universe," in either model, suppose we neglect the action of the perturbation $\sim g$ over the interval (t, t_0). Then, the off-diagonal components of the density matrix decay according to the solution of Eq.(15),

$$\langle n|\rho(t)|m\rangle = e^{-(t-t_0)[i\epsilon(n-m)+\frac{\lambda}{2}(n-m)^2]}\langle n|\rho(t_0)|m\rangle. \quad (22)$$

Of course, the density matrix approaching diagonal form does not guarantee that each individual state vector in the ensemble is approaching some definite state $|n\rangle$, but that is the case here[2].

However, one cannot neglect the action of the perturbation: it is working alongside of the collapse. As we have seen in section 2, when collapse competes with the Hamiltonian evolution, the outcome is not obvious.

What is needed if the collapse is to "choose a universe?". A universe is a macroscopic object and, as such, should have well-defined values for macroscopic variables such as the amount of mass it contains. That is, for any probable state vector in the ensemble, although it is a superposition of states of various n-values, its range of n-values should be small compared to its mean n-value. Typically, this requirement is quantified by requiring the deviation from the mean to be small compared to the mean.

Let's state this requirement in the context of our problem. For any operator A, define its expectation value as

$$\langle A\rangle(t) \equiv {}_w\langle\Psi, t|A|\Psi, t\rangle_w/{}_w\langle\Psi, t|\Psi, t\rangle_w.$$

Define the ensemble average of products of such expectation values as

$$\overline{\langle A\rangle\langle B\rangle...} \equiv \int Dw_w\langle\Psi, t|\Psi, t\rangle_w\langle A\rangle\langle B\rangle....$$

Then, the squared standard deviation of N is

$$(\Delta N)^2 \equiv \overline{\langle(N - \langle N\rangle)^2\rangle} = \overline{\langle N^2\rangle} - \overline{\langle N\rangle^2},$$

and the requirement is $\Delta N/\overline{\langle N\rangle} << 1$.

It should be emphasized that $(\Delta N)^2$ is *not*

$$(\Delta_0 N)^2 \equiv \overline{\langle(N - \overline{\langle N\rangle})^2\rangle} = \overline{\langle N^2\rangle} - \overline{\langle N\rangle}^2.$$

Indeed, $(\Delta N)^2 \leq (\Delta_0 N)^2$, since

$$0 \leq \overline{[\langle N - \overline{\langle N\rangle}\rangle]^2} = \overline{\langle N\rangle^2} - \overline{\langle N\rangle}^2.$$

As we have seen, using the density matrix it is easy to calculate ensemble averages involving two state vectors. However, it is not easy to calculate ensemble averages when four or more state vectors are involved.

In the two previous sections we have shown how to calculate $\overline{\langle N\rangle}(t)$, and can similarly calculate $\overline{\langle N^2\rangle}(t)$, so $(\Delta_0 N)^2$ is easy to find. It turns out that, for large t, for both models, $(\Delta_0 N)^2 \sim \overline{\langle N\rangle}^2$ (i.e., $\sim t^2$ for the first and $\sim \exp 2st$ for the second).

However, $\overline{\langle N\rangle^2(t)}$ involves the ensemble average of a product of four state vectors, so $(\Delta N)^2$ is not easy to find.

Here is what we have been able to do. In Appendix C, it is shown in Eq.(C9) that

$$\begin{aligned}\frac{d}{dt}(\Delta N)^2 &= \frac{d}{dt}(\Delta_0 N)^2 \\ &+2i\overline{\langle[N - \overline{\langle N\rangle}, H]\rangle\langle N - \overline{\langle N\rangle}\rangle} \\ &-4\lambda\overline{\langle(N - \langle N\rangle)^2\rangle\langle(N - \langle N\rangle)^2\rangle}. \quad (23)\end{aligned}$$

This would seem to increase the difficulty since, e.g., the last term in (23) is the sum of terms involving the average of products of four, six and eight state vectors. However let us make the not unreasonable assumption that, for most state vectors of high probability, $\langle N - \overline{\langle N\rangle}\rangle$ is nearly constant, and likewise for $\langle(N - \langle N\rangle)^2\rangle$. Then

$$\overline{\langle A\rangle\langle N - \overline{\langle N\rangle}\rangle} \approx \overline{\langle A\rangle}\,\overline{\langle N - \overline{\langle N\rangle}\rangle} = 0,$$

$$\overline{\langle(N - \langle N\rangle)^2\rangle\langle(N - \langle N\rangle)^2\rangle} \approx [\overline{\langle(N - \langle N\rangle)^2\rangle}]^2,$$

and Eq.(23) becomes

$$\frac{d}{dt}(\Delta N)^2 = \frac{d}{dt}(\Delta_0 N)^2 - 4\lambda[(\Delta N)^2]^2. \quad (24)$$

This is an example of the Riccati equation $dz/dt + z^2 = f(t)$. It may be converted to a linear second order equation by the transformation $z = \dot{u}/u$, its solution for our two examples for large t involves Bessel functions, etc. However, the important point is that, for our two examples for large t, the left side of Eq.(24) is negligible compared to either of the two terms on the right side, so

$$(\Delta N)^2 \approx \frac{1}{2}\sqrt{\frac{d}{\lambda dt}(\Delta_0 N)^2}. \quad (25)$$

For the example in section III, we may write, $(\Delta_0 N)^2 \approx (N_0 t/T_0)^2$ for large t, where N_0 is the current number of Planck masses in the universe and T_0 is the current age of the universe. We therefore find from Eq.(25) that

$$(\Delta N)^2 \approx \frac{N_0}{\sqrt{2\lambda T_0}} \text{ and } \frac{\Delta N}{\overline{\overline{\langle N \rangle}}} \approx \frac{1}{[2\lambda T_0]^{1/4} N_0^{1/2}}.$$

For the example in section IV, we may write, $(\Delta_0 N)^2 \approx [N_0 \exp s_0(t - T_0)]^2$ for large t, where $s_0 N_0$ is the current rate of increase of Planck masses (see the next section for more discussion of this). We therefore find from Eq.(25) that

$$(\Delta N)^2 \approx \frac{N_0}{\sqrt{2\lambda/s_0}} \text{ and } \frac{\Delta N}{\overline{\overline{\langle N \rangle}}} \approx \frac{1}{[2\lambda/s_0]^{1/4} N_0^{1/2}}.$$

As mentioned in the next section, it is plausible that λT_0 and λ/s_0 are of order 1. Therefore, indeed, $\Delta N/\overline{\langle N \rangle} \approx N_0^{-1/2} << 1$.

VI. CONCLUDING REMARKS

By means of a simple quantum mechanical model, the usefulness of collapse dynamics in creating the universe out of nothing after a long wait, generating its energy, and choosing a macroscopic universe out of a superposition, has been sketched. The model may be regarded as just an illustration, but one might try to take it more seriously.

It is perhaps natural to choose λ^{-1} and g^{-1} to be of the order of the Planck time T_p. If one takes the volume of the present universe to be $V_0 \approx 3 \times 10^{80}$ cubic meters, takes the mass-energy density to be the present critical density value $\rho_0 \approx 10^{-26}$kg/m^3 then, with the Planck mass $\epsilon = M_p \approx 2 \times 10^{-8}$kg, there are presently $N_0 = \rho_0 V_0/M_p \approx 10^{62}$ Planck masses in the universe.

Suppose that all this mass-energy is generated at the beginning of the universe, according to the model of section IV, $N(t) \approx \exp(t/T_p)$. It would take time $t \approx 140T_p$ to generate N_0 such masses. If each Planck mass occupies a Planck volume, the universe at that time would be of radius $\approx (3N_0/4\pi)^{1/3} L_p \approx 5 \times 10^{-13}$cm ($L_p \approx 1.6 \times 10^{-33}$cm is the Planck length), about 5 times larger than the radius of a proton. Of course, with this scenario, one wants the process to abruptly stop at this time. One might make g, λ and ϵ proportional to $\Theta(N_0 - N)$, and let a different Hamiltonian take over the subsequent evolution but this seems rather artificial.

More interesting would be if the process were ongoing, with λ a suitably decreasing function of N, to provide a mechanism which continually slows the energy production down, yielding the correct amount of energy and energy production rate at the present era.

With regard to the current energy production rate, here is a suggestive result.

According to the model, given the present time T_0, the number of Planck masses at time $t \geq T_0$ is $N(t) = N_0 \exp \lambda_0 (t - T_0)$, where λ_0 is the current collapse rate. Thus, since the energy in the universe is $E(t) = M_p N(t)$, its present rate of change is

$$dE/dt = E_0 \lambda_0.$$

On the other hand, suppose next that the radius of the universe is expanding with the speed of light, $dR/dt = c$. Then, the present rate of increase of energy in the universe is

$$dE/dt = \rho_0 dV/dt = \rho_0 4\pi R_0^2 c = 3E_0 c/R_0.$$

Equating these two expressions for dE/dt, using $R_0 \approx (3V_0/4\pi)^{1/3} = 4 \times 10^{26}$m, we obtain

$$\lambda_0 = 3c/R_0 \approx 2 \times 10^{-18}\text{sec}^{-1},$$

approximately the inverse of the age of the universe.

This is not far from the value $\lambda \approx 10^{-16}$sec^{-1} suggested for the GRW theory[7] and adopted for CSL. It suggests considering the consequences if the collapse rate in CSL, hitherto considered a constant is, instead, the inverse of the age of the universe. This would imply a large rate of excitation of matter at the beginning of the universe. However, it would not have much effect, either on visible excitation or on the collapse of macroscopic objects over a time interval, say, 1/100 to 100 times the present age of the universe.

The reader will have noted that there has been no mention of general relativity. Connection with the FRW formalism can be made through identifying an operator for the "radius R" of the universe.

R depends not only on the operator $N(t)$ but also on the operator representing the volume $v(t)$ of a cell. v must have dynamics because, if it equals the Planck volume at the beginning of the universe, it does not remain so. The current value of v may be found as follows. The change of V is $dV = vdN$, while the change of energy is $dE = \epsilon dN$. Since $dE/dV = \rho_0$, we obtain $v = \epsilon/\rho_0 \approx 2 \times 10^{18}$m^3, i.e., cells of Planck energy which are currently added have length ≈ 1000km.

So, there must be an addition to the Hamiltonian to give dynamics for the operator v, presumably guided by the FRW formalism, with

$$\frac{4\pi}{3} R^3(t) = \int_0^t dt' \frac{dN(t')}{dt'} v(t').$$

Moreover, if the created energy is to include not only dark energy but also visible matter and dark matter, dynamics must be included describing how the latter two evolve from the total energy $N\epsilon$.

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Appendix A: Spin Problem

Consider an initially normalized state vector $|\Psi, t\rangle$ describing a spin in the $x-z$ plane. If $0 \leq \theta \leq 2\pi$ is the angle between the $z-$axis and the spin vector, the components of the spin in the σ_z basis are $\langle\uparrow|\Psi, t\rangle = \cos(\theta/2)$, $\langle\downarrow|\Psi, t\rangle = \sin(\theta/2)$ (up to a phase factor). The spin part of the evolution given in Eq. (1), for an infinitesimal time interval dt, is:

$$|\Psi, t+dt\rangle_w = e^{-dt\{\frac{i}{2}\omega\sigma_2 + \frac{1}{4\lambda}[w(t)-2\lambda\sigma_1]^2\}}|\Psi, t\rangle_w. \quad \text{(A1)}$$

Eq. (A1), expressed in the σ_z basis, retaining all terms necessary for first order in dt is:

$$\begin{aligned} A\cos[\theta(t+dt)/2] &= e^{-\frac{1}{4\lambda}w^2(t)}\{\cos(\theta/2)[1+w(t)dt \\ &+\frac{1}{2}(w(t)dt)^2 - \lambda dt] - \sin(\theta/2)[\frac{1}{2}\omega dt]\} \\ A\sin[\theta(t+dt)/2] &= e^{-\frac{1}{4\lambda}w^2(t)}\{\sin(\theta/2)[1-w(t)dt \\ &+\frac{1}{2}(w(t)dt)^2 - \lambda dt] + \cos(\theta/2)[\frac{1}{2}\omega dt]\} \end{aligned} \quad \text{(A2)}$$

The amplitude A is necessary because the state vector evolution is not unitary. Indeed, this is what makes the probability rule Eq. (2) meaningful:

$$\begin{aligned} &P(w(t)) \sim {}_w\langle\Psi, t|\Psi, t\rangle_w = A^2 \\ &= e^{-\frac{1}{2\lambda}w^2(t)}\{1 + 2w(t)dt[\cos^2(\theta/2) - \sin^2(\theta/2)] \\ &+2[\frac{1}{2}(w(t)dt)^2 - \lambda dt] + (w(t)dt)^2\} \\ &= e^{-\frac{1}{2\lambda}w^2(t)}[1 + 2w(t)dt\cos\theta] \end{aligned} \quad \text{(A3)}$$

We note that the total probability is 1, as may be seen by multiplying Eq. (A3) by $(2\pi\lambda/dt)^{-1/2}dw(t)$ and integrating over $(-\infty, \infty)$. In the last step of Eq. (A3), we have set $(w(t)dt)^2 = \lambda dt$, since that is what its integral gives to order dt in all of our calculations (the same thing could have been done in Eq. (A2) and shall be done in subsequent equations).

Solving Eqs. (A2) to order dt yields:

$$\begin{aligned} \theta(t+dt) &= 2\tan^{-1}\{\tan(\theta/2) + \frac{1}{2}\omega dt\sec^2(\theta/2) \\ &+ 2[\lambda dt - w(t)dt]\tan(\theta/2)\} \end{aligned} \quad \text{(A4)}$$

Using $\tan^{-1}(\tan\phi + x) = \phi + x\cos^2\phi - x^2\sin\phi\cos^3\phi + ...$ in Eq. (A4) results in:

$$d\theta(t) = \omega dt - 2w(t)dt\sin\theta + 2\lambda dt\sin\theta\cos\theta \quad \text{(A5)}$$

Now that we have Eqs.(A3, A5), we can find $\overline{d\theta}$, $\overline{(d\theta)^2}$, which are used in constructing the Fokker-Planck equation for the probability distribution of θ in section II:

$$\begin{aligned} \overline{d\theta} &\equiv \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi\lambda/dt}}dw(t)P(w(t))d\theta(t) \\ &= \omega dt - 2\lambda dt\sin\theta\cos\theta \quad \text{(A6)} \\ \overline{(d\theta)^2} &\equiv \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi\lambda/dt}}dwP(w(t))(d\theta)^2(t) \\ &= 4\lambda dt\sin^2\theta. \quad \text{(A7)} \end{aligned}$$

Appendix B: Solution of Eq.(4) for $\omega = 0$

For convenience, we give here the solution[5] of

$$\frac{\partial}{\partial t}R = 2\lambda\Big\{\frac{\partial}{\partial\theta}[\sin\theta\cos\theta]R + \frac{\partial^2}{\partial\theta^2}\sin^2\theta R\Big\}. \quad \text{(B1)}$$

By changing variables to the squared amplitude $x = \cos^2(\theta/2)$, and $R = (1/2)\sin\theta R'$, the equivalent equation is obtained:

$$\frac{\partial}{\partial t}R' = 8\lambda\frac{\partial^2}{\partial x^2}[x(1-x)]^2R'. \quad \text{(B2)}$$

where $0 \leq x \leq 1$. Eq.(B2) clearly shows the no-drift condition $d\overline{x}(t)/dt = 0$ necessary for Born-rule-obeying collapse, as discussed in section II A. It also shows how the diffusion vanishes at $x = 0, 1$, providing absorbing boundary conditions.

A further change of variables, $z = \ln[x/(1-x)]$ and $R' = x(1-x)R''$ provides an equation with normal diffusion,

$$\frac{\partial}{\partial t}R'' = 8\lambda\Big\{\frac{\partial^2}{\partial z^2}R'' - \frac{\partial}{\partial z}\tanh(z/2)R''\Big\}, \quad \text{(B3)}$$

which is readily solved:

$$\begin{aligned} R''(z) = \frac{1}{8\sqrt{2\pi\lambda t}\cosh(z_0/2)}\Big\{&e^{z_0/2}e^{\frac{-1}{32\lambda t}[z-z_0-8\lambda t]^2} \\ &+e^{-z_0/2}e^{\frac{-1}{32\lambda t}[z-z_0+8\lambda t]^2}\Big\}. \end{aligned} \quad \text{(B4)}$$

We see from Eq.(B4) that $R''(0) \to \delta(z - z_0)$ and

$$R''(\infty) \to \frac{1}{\cosh(z_0/2)}\Big\{e^{z_0/2}\delta(z-\infty) + e^{-z_0/2}\delta(z+\infty)\Big\}.$$

In terms of θ, the solution (B4) of Eq.(B1) is

$$\begin{aligned} &R(\theta)d\theta = \frac{1}{\sqrt{8\pi\lambda}}d\ln\tan(\theta/2) \\ &\Big\{\cos^2(\theta_0/2)e^{\frac{-1}{8\lambda t}\left[\ln\frac{\tan(\theta/2)}{\tan(\theta_0/2}+4\lambda t\right]^2} \\ &\quad+\sin^2(\theta_0/2)e^{\frac{-1}{8\lambda t}\left[\ln\frac{\tan(\theta/2)}{\tan(\theta_0/2}-4\lambda t\right]^2}\Big\}. \end{aligned} \quad \text{(B5)}$$

We see from Eq.(B5) that $R(0) \to \delta(\theta - \theta_0)$ and note the collapse behavior obeying the Born rule,

$$R''(\infty) \to \cos^2(\theta_0/2)\delta(\theta) + \sin^2(\theta_0/2)\delta(\theta - \pi).$$

Appendix C: Ensemble Averages of Products

We first prove the following theorem. Consider the state vector evolution (13). Then

$$\begin{aligned} \frac{d}{dt}\overline{\langle A\rangle\langle B\rangle} &= \overline{\langle -i[A,H] - (\lambda/2)[N,[N,A]]\rangle\langle B\rangle} \\ &+\overline{\langle A\rangle\langle -i[B,H] - (\lambda/2)[N,[N,B]]\rangle} \\ &+4\lambda\overline{\langle N\rangle^2\langle A\rangle\langle B\rangle} \\ &-2\lambda[\overline{\langle N\rangle\langle A\rangle\langle BN+NB\rangle} + \overline{\langle N\rangle\langle B\rangle\langle AN+NA\rangle}] \\ &+\lambda[\overline{\langle AN+NA\rangle\langle BN+NB\rangle}]. \end{aligned} \quad \text{(C1)}$$

In Eq.(C1), $\langle Q\rangle(t) \equiv_w\langle\Psi,t|Q|\Psi,t\rangle_w/_w\langle\Psi,t|\Psi,t\rangle_w$. The overline refers to the ensemble average: for example, $\overline{\langle A\rangle}(t)=\int Dw_w\langle\Psi,t|\Psi,t\rangle_w\langle A\rangle(t)\equiv\text{Tr}A\rho(t)$, where $\rho(t)$ is the density matrix.

According to Eq.(13), the state vector at time $t+dt$ is

$$|\Psi,t+dt\rangle_{w',w}=e^{-idtH-\frac{dt}{4\lambda}[w'-2\lambda N]^2}|\Psi,t\rangle_w=e^{-\frac{1}{4\lambda}dtw'^2}\cdot[1-idtH+dtw'N-(\lambda/2)dtN^2]|\Psi,t\rangle_w. \quad \text{(C2)}$$

In Eq.(C2), we have written w' as the scalar field at time $t+dt$, and w represents the scalar field for $0\leq t\leq t$. We have also set $(w'dtN)^2/2=\lambda dtN^2/2$, and will continue to do so, since that is what it will equal when integrated over the probability.

The ensemble average of the expectation value of A at time $t+dt$ (calculated using Eq.(C2)), is obtained by multiplying the expectation value,

$$\langle A\rangle(t+dt)=\langle A\rangle(t)+dt\{-i\langle[A,H]\rangle(t)+w'\langle\{N,A\}_+\rangle(t)-(\lambda/2)\langle[N,[N,A(t)]]\rangle(t)\}\cdot\Big\{\frac{{}_w\langle\Psi,t|\Psi,t\rangle_w}{{}_w\langle\Psi,t|(1+2dtw'N)|\Psi,t\rangle_w}\Big\} \quad \text{(C3)}$$

by the probability density

$${}_{w',w}\langle\Psi,t+dt|\Psi,t+dt\rangle_{w',w}=e^{-\frac{1}{2\lambda}dtw'^2}\cdot{}_w\langle\Psi,t|e^{2dtw'N-2\lambda N^2}|\Psi,t\rangle_w=e^{-\frac{1}{2\lambda}dtw'^2}\,{}_w\langle\Psi,t|(1+2dtw'N)|\Psi,t\rangle_w \quad \text{(C4)}$$

and by $dw'Dw$, and integrating over all w',w, to obtain the well-known result:

$$\frac{d}{dt}\overline{\langle A\rangle(t)}=\text{Tr}\Big[\{-i[A,H]-(\lambda/2)[N,[N,A(t)]]\}\rho(t)\Big] \quad \text{(C5)}$$

Similarly, the ensemble average of the product of two density matrices at time $t+dt$ is

$$\overline{\langle A\rangle(t+dt)\langle B\rangle(t+dt)}=\int dw'Dwe_w^{-\frac{1}{2\lambda}dtw'^2}\langle\Psi,t|\Psi,t\rangle_w\cdot\Big\{\langle A\rangle(t)+dt\{-i\langle[A,H]\rangle(t)+w'\langle\{N,A\}_+\rangle(t)-(\lambda/2)\langle[N,[N,A(t)]]\rangle(t)\}\Big\}\cdot\Big\{\langle B\rangle(t)+dt\{-i\langle[B,H]\rangle(t)+w'\langle\{N,B\}_+\rangle(t)-(\lambda/2)\langle[N,[N,B(t)]]\rangle(t)\}\Big\}\cdot\Big\{\frac{{}_w\langle\Psi,t|\Psi,t\rangle_w}{{}_w\langle\Psi,t|(1-2dtw'N)|\Psi,t\rangle_w}\Big\}. \quad \text{(C6)}$$

Writing the last bracket in Eq.(C6) as

$$\Big\{\frac{{}_w\langle\Psi,t|\Psi,t\rangle_w}{{}_w\langle\Psi,t|(1-2dtw'N)|\Psi,t\rangle_w}\Big\}=1+2dtw'\langle N\rangle(t)+4\lambda dt\langle N\rangle^2(t), \quad \text{(C7)}$$

the integrals can be performed with the result Eq.(C1).

One may check that, if $B=1$ is inserted into Eq.(C1), then Eq.(C5) is obtained.

Now, set $A=B=N$. Eq.(C1) becomes:

$$\frac{d}{dt}\overline{\langle N\rangle^2(t)}=-2i\overline{\langle[N,H]\rangle\langle N\rangle}+4\lambda\overline{\langle(N-\langle N\rangle)^2\rangle\langle(N-\langle N\rangle)^2\rangle}. \quad \text{(C8)}$$

Since

$$\overline{\langle(N-\langle N\rangle)^2\rangle}=\overline{\langle N^2\rangle}-\overline{\langle N\rangle^2}$$

and

$$\overline{\langle(N-\overline{\langle N\rangle})^2\rangle}=\overline{\langle N^2\rangle}-\overline{\langle N\rangle}^2,$$

it follows from Eq.(C8) that

$$\frac{d}{dt}\overline{\langle(N-\langle N\rangle)^2\rangle}=\frac{d}{dt}\overline{\langle(N-\overline{\langle N\rangle})^2\rangle}+2i\overline{\langle[N-\overline{\langle N\rangle},H]\rangle\langle(N-\overline{\langle N\rangle}\rangle}-4\lambda\overline{\langle(N-\langle N\rangle)^2\rangle\langle(N-\langle N\rangle)^2\rangle}. \quad \text{(C9)}$$

which is used in section V, Eq.(23).

[1] P. Pearle, Phys. Rev. A**39**, 2277 (1989): G. C. Ghirardi, P. Pearle and A. Rimini, Phys. Rev. A**42**, 78 (1990).

[2] Some reviews are: P. Pearle in *Open systems and measurement in relativistic quantum theory*, F. Petruccione and H. P. Breuer eds., p.195 (Springer Verlag, Heidelberg 1999): A. Bassi and G. C. Ghirardi, *Physics Reports* **379**, 257 (2003): P. Pearle, Journ. Phys. A: Math Theor. 40, 3189 (2007) and continued in *Quantum Reality, Relativistic Causality and Closing the Epistemic Circle*, eds. W. Myrvold and J. Christian (Springer, 2009), pp.257-292.

[3] It is a remarkable property of CSL that, because the collapse rate of particles of mass m is governed by their mass density operator (as opposed to the number density operator multiplied by something other than m), the energy production rate is quite small: see P. Pearle and E. Squires, Phys. Rev. Lett **73**, 1 (1994). Verification/refutation of CSL may come about due to future highly sensitive experiments which detect/do not detect such an energy increase.

[4] A. Bassi and E. Ippoliti, Phys. Rev. **A69**, 012105 (2004)

[5] P. Pearle, Phys. Rev. **D13**, 857 (1976).

[6] P. Pearle, Journ. of Stat. Phys. **41**, 719 (1985).

[7] G. C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D**34**, 470 (1986).